

Resonant conversions of QCD axions into hidden axions and suppressed isocurvature perturbations

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Abstract

We study in detail MSW-like resonant conversions of QCD axions into hidden axions, including cases where the adiabaticity condition is only marginally satisfied, and where anharmonic effects are non-negligible. When the resonant conversion is efficient, the QCD axion abundance is suppressed by the hidden and QCD axion mass ratio. We find that, when the resonant conversion is incomplete due to a weak violation of the adiabaticity, the CDM isocurvature perturbations can be significantly suppressed, while non-Gaussianity of the isocurvature perturbations generically remain unsuppressed. The isocurvature bounds on the inflation scale can therefore be relaxed by the partial resonant conversion of the QCD axions into hidden axions.

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I. INTRODUCTION

The strong CP problem in the standard model is one of the most profound mysteries in particle physics, and a plausible solution is the Peccei-Quinn (PQ) mechanism [1]. When a global $U(1)_{\text{PQ}}$ symmetry is spontaneously broken, there appears an associated Nambu-Goldstone (NG) boson “axion” which is assumed to acquire a tiny mass predominantly from the QCD anomaly [2–4]. As a result, the axion is stabilized at the CP-conserving minimum, and the strong CP problem is dynamically solved.

In the early Universe, the QCD axion remains massless until the cosmic temperature drops down to the QCD scale, $\Lambda_{\text{QCD}} \simeq 400 \text{ MeV}$. During the QCD phase transition, the QCD axion gradually acquires a mass, and it starts to oscillate about the CP conserving minimum when the mass becomes comparable to the Hubble parameter. The PQ mechanism is therefore necessarily accompanied by coherent oscillations of the QCD axions, which behave as cold dark matter (CDM). The abundance of axion coherent oscillations is given by [5]

$$\Omega_a h^2 \simeq 0.195 \theta_i^2 f(\theta_i) \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.184}, \quad (1)$$

where θ_i is the initial misalignment angle of the QCD axion, F_a is the QCD axion decay constant, and $f(\theta_i)$ is the anharmonic correction that is a monotonically increasing function of θ_i : $f(\theta_i) \sim 1$ for $\theta_i \lesssim 1$ and it grows rapidly as θ_i approaches π [6].

There are two possible cosmological problems of the QCD axion dark matter. One is the overabundance; if the decay constant is of order the GUT or string scale, the axion abundance would be many orders of magnitude larger than the observed dark matter unless the initial misalignment angle is less than 10^{-2} . The other is the tight isocurvature constraint on the inflation scale; if the axion is present during inflation, it acquires quantum fluctuations of order the Hubble parameter, which induce the CDM isocurvature perturbations. The upper bound on the inflation scale is so tight that it excludes large-field inflation models [7–9].¹

¹ There have been proposed various ways to suppress the axion CDM isocurvature perturbations [7, 10–17].

The QCD axion may not be the only pseudo NG boson in nature; there may be many axions or axion-like particles (ALPs) [18–22]. Indeed, in a certain class of the compactification of the extra dimensions, there remain many light axions, some of which may play an important cosmological role [23]. For instance, multiple axions in cosmological contexts have been studied in the so-called axiverse scenario [24, 25] and the axion landscape [26, 27]. Suppose that there is another axion which has a mixing with the QCD axion. Then, as the QCD axion gradually acquires a mass during the QCD phase transition, the MSW-like conversions could take place. Such resonant conversion was studied in Ref. [28] assuming that adiabaticity condition is satisfied and anharmonic effects are negligible.

In this letter we study in detail the resonant conversions of axions, including cases where the adiabaticity condition is only marginally satisfied or weakly violated, and where the anharmonic effects are non-negligible. We show that the axion abundance can be suppressed by the mass ratio of the hidden and QCD axions. This is because it is the number of the axions in a comoving volume that is conserved during the resonant conversion process. The authors of Ref. [28] claimed that the QCD axion abundance is suppressed by the square of the mass ratio because the oscillation amplitude is conserved in the conversion process, which however was not confirmed in our analysis. We shall also study how the isocurvature perturbations are modified when the resonant conversions take place. Interestingly, we find that, when the resonant conversion is incomplete due to weak violation of the adiabaticity condition, the power spectrum of the isocurvature perturbations can be significantly suppressed for certain parameters. This is because the conversion rate also depends on the initial misalignment angle, and the produced hidden axions can compensate the isocurvature perturbations of the QCD axions. Therefore, the isocurvature constraint on the inflation scale can be relaxed by the incomplete resonant conversions. On the other hand, non-Gaussianity of isocurvature perturbations are generically non-vanishing even in this case.

The rest of this letter is organized as follows. In Sec. II, we give our model of axions and study the dynamics of the axion oscillations, focusing on the resonant conversion

processes. In Sec. III, we show how the axion abundance and isocurvature perturbations are modified by the resonant conversion. Sec. IV is devoted to conclusions and discussions.

II. SET-UP

In this section we give a model of the QCD and hidden axions which have a non-zero mixing. In the next section we study the dynamics of axion coherent oscillations, focusing on the resonant conversion between these two axions.

Let us introduce two complex scalar fields Φ and Φ_H with the following interactions with heavy quarks,

$$\mathcal{L} = \kappa \Phi Q \bar{Q} + \frac{\lambda}{M_P} \Phi \Phi_H Q_H \bar{Q}_H \quad (2)$$

where Q and \bar{Q} belong to $\mathbf{3} + \bar{\mathbf{3}}$ of $SU(3)_c$, Q_H and \bar{Q}_H belong to (anti-)fundamental representation $\mathbf{N} + \bar{\mathbf{N}}$ of a hidden gauge symmetry $SU(N)_H$, and κ and λ are dimensionless coupling constants. $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. This is an extension of the KSVZ hadronic axion model with additional hidden scalar and quarks. We assume that there are two global $U(1)$ symmetries, $U(1)_{PQ}$ and $U(1)_H$, which are spontaneously broken by vacuum expectation values of Φ and Φ_H . See Table I for the charge assignment of each field. The QCD axion (a) and the hidden axion (a_H) appear as (pseudo) NG bosons associated with the spontaneous symmetry breaking, and they reside in the phase component of Φ and Φ_H , respectively. We assume that the hidden gauge symmetry $SU(N)_H$ becomes strong and induces a potential for axions in the low energy. If the hidden gauge sector is not thermalized by the inflaton decay, the axion potential is generated when the Hubble parameter becomes comparable to the dynamical scale.

The low-energy effective Lagrangian for axions below the dynamical scale of $SU(N)_H$ is given by

$$\mathcal{L} = \frac{1}{2} \partial^\mu a \partial_\mu a + \frac{1}{2} \partial^\mu a_H \partial_\mu a_H - V(a, a_H) \quad (3)$$

with the potential,

$$V(a, a_H) = m_a^2(T) F_a^2 \left[1 - \cos \left(\frac{a}{F_a} \right) \right] + m_H^2 F_H^2 \left[1 - \cos \left(\frac{a_H}{F_H} + \frac{a}{F_a} \right) \right], \quad (4)$$

	Φ	Φ_H	Q	\bar{Q}	Q_H	\bar{Q}_H
U(1) _{PQ}	1	0	-1	0	-1	0
U(1) _H	0	1	0	0	-1	0

TABLE I: U(1)_{PQ} and U(1)_H charge assignment.

where $m_a(T)$ and m_H are the mass of the QCD and hidden axions respectively, F_a and F_H are the decay constants of a and a_H , and they are comparable to the corresponding U(1) symmetry breaking scales.² Here we have shifted a and a_H so that the origins of a and a_H coincide with the potential minimum. Then, the equations of motion for the axions are given by

$$\begin{aligned} \ddot{a} + 3H\dot{a} + \frac{m_H^2 F_H^2}{F_a} \sin\left(\frac{a_H}{F_H} + \frac{a}{F_a}\right) + m_a^2(T) F_a \sin\left(\frac{a}{F_a}\right) &= 0 \\ \ddot{a}_H + 3H\dot{a}_H + m_H^2 F_H \sin\left(\frac{a_H}{F_H} + \frac{a}{F_a}\right) &= 0. \end{aligned} \quad (6)$$

If the axions are initially located in the vicinity of the potential minimum, or if the oscillation amplitudes become much smaller than the corresponding decay constant, the equations of motion can be approximately linearized as

$$\ddot{A} + 3H\dot{A} + M^2 A \approx 0 \quad (7)$$

where A and M^2 are respectively the column vector of two axion fields and the (squared) mass matrix,

$$A = \begin{pmatrix} a \\ a_H \end{pmatrix} \quad \text{and} \quad M^2 = \begin{pmatrix} m_a^2(T) + \left(\frac{F_H}{F_a}\right)^2 m_H^2 & \mu^2 \\ \mu^2 & m_H^2 \end{pmatrix}, \quad (8)$$

² It is possible to generalize the potential as

$$V(a, a_H) = m_a^2(T) F_a^2 \left[1 - \cos\left(n_1 \frac{a_H}{F_H} + n_2 \frac{a}{F_a}\right) \right] + m_H^2 F_H^2 \left[1 - \cos\left(n_3 \frac{a_H}{F_H} + n_4 \frac{a}{F_a}\right) \right], \quad (5)$$

with $n_1 - n_4$ being some integers, for appropriate charge assignments. In the text we focus on the case of $n_1 = 0$ and $n_2 = n_3 = n_4 = 1$, but we can straightforwardly extend our analysis to more general cases.

and we define $\mu^2 = (F_H/F_a)m_H^2$. One can diagonalize the mass matrix by an orthogonal matrix O as

$$\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = O^T M^2 O \quad \text{and} \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = O^T A \quad (9)$$

with $m_2 > m_1$. In our convention, the mass of a_2 is always heavier than or equal to that of a_1 . Alternatively, one may define the mass eigenstates by

$$\begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} a \\ a_H \end{pmatrix} \quad (10)$$

with the mixing angle

$$\tan 2\alpha = -\frac{2\mu^2}{m_a^2(T) + [(F_H/F_a)^2 - 1]m_H^2}. \quad (11)$$

Throughout this letter we use the previous notation of the mass eigenstates (a_1, a_2) with $m_2 > m_1$ which is more convenient when the resonant conversion takes place.

III. COSMOLOGY OF AXION RESONANT CONVERSION

A. Cross-over of mass eigenvalues

The QCD axion remains almost massless at temperatures much higher than the QCD dynamical scale $\Lambda_{\text{QCD}} \simeq 400 \text{ MeV}$, and it gradually acquires a mass and starts to oscillate during the QCD phase transition. The temperature-dependent QCD axion mass is given by [29]³

$$m_a(T) \approx \begin{cases} 4.05 \times 10^{-4} \frac{\Lambda_{\text{QCD}}^2}{F_a} \left(\frac{T}{\Lambda_{\text{QCD}}} \right)^{-3.34} & \text{for } T > 0.26\Lambda_{\text{QCD}} \\ 3.82 \times 10^{-2} \frac{\Lambda_{\text{QCD}}^2}{F_a} & \text{for } T < 0.26\Lambda_{\text{QCD}} \end{cases}, \quad (12)$$

where the typical time scale over which the QCD axion mass grows is the Hubble time H^{-1} . Therefore, if the zero-temperature mass of the QCD axion, $m_a \equiv m_a(T=0)$, is heavier

³ Precisely speaking, $m_a(T)$ parametrizes the potential height in Eq. (4), and the actual mass eigenvalue is slightly different due to the mixing with the hidden axion.

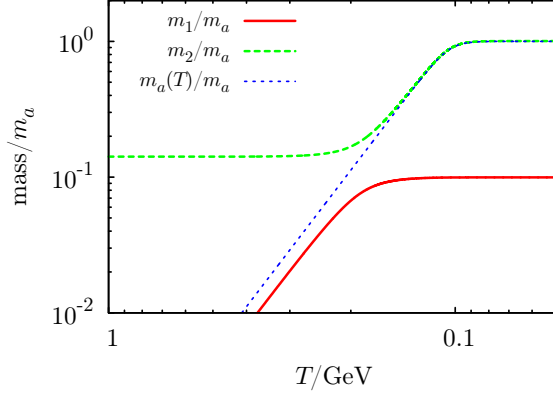


FIG. 1: Evolution of mass eigenvalues as a function of temperature T . The masses are normalized by the zero-temperature QCD axion mass $m_a = m_a(T = 0)$, and the temperature is in the GeV units. The solid (red) and dashed (green) lines represent evolution of the light (m_1) and heavy (m_2) mass eigenvalues, respectively, while the dotted (blue) line represents the temperature-dependent QCD axion mass, $m_a(T)$. We have taken $F_H = F_a = 10^{14}$ GeV and $m_H = 0.1 m_a$.

than the hidden axion mass, there is necessarily a cross-over of the mass eigenvalues. The resonance temperature T_{res} is given by the temperature at which the QCD axion mass becomes equal to the hidden axion mass:

$$T_{\text{res}} \simeq 0.1 \left(\frac{\Lambda_{\text{QCD}}^2}{F_a m_H} \right)^{0.3}, \quad (13)$$

where $m_H < m_a$ is assumed. See Fig. 1 for the typical evolution of the mass eigenvalues as a function of the cosmic temperature T . We can see that a_1 and a_2 are initially identified as the QCD and hidden axion respectively for $T > T_{\text{res}}$, and eventually they are exchanged with each other after the cross-over of the mass eigenvalues.

At temperature $T \approx T_{\text{res}}$, QCD axions are converted to hidden axions through resonance *a la* the MSW effect in neutrino physics [30], and vice versa. This conversion process occurs efficiently if the adiabaticity condition is satisfied, namely, if both axions oscillate many times over the Hubble time. Then, there is an adiabatic invariant during the resonant conversion. In particular, if the axion potentials are approximated by the

quadratic potential, it is the number of axions in a co-moving volume that is conserved, not the amplitude of oscillations. The hidden axion starts to oscillate when the Hubble parameter becomes comparable to its mass, $H \sim m_H$, unless it is initially located in the vicinity of the potential maximum. The oscillation amplitude of a_H is likely so small at the resonance temperature that its dynamics can be approximated by the harmonic oscillations. To parametrize the adiabaticity, we define a parameter ξ as a ratio of the Hubble parameter to the axion mass evaluated at the resonance:

$$\xi \equiv \frac{H(T_{\text{res}})}{m_H} = \frac{H(T_{\text{res}})}{m_a(T_{\text{res}})} \simeq 4.4 \left(\frac{m_H}{m_a} \right)^{-1.6} \frac{F_a}{M_P}. \quad (14)$$

Note that the hidden axion mass is equal to the QCD axion mass at the cross-over temperature. As we shall see shortly, the resonance occurs efficiently for $\xi \ll 1$, and becomes incomplete as ξ approaches unity. This sets the lower bound on m_H for the efficient resonant conversion; this bound roughly reads $m_H \gtrsim H(T = \Lambda_{\text{QCD}}) \simeq 2 \times 10^{-10} \text{ eV}$. The above adiabaticity parameter does not take account of the anharmonic effects, which become important when the initial misalignment angle θ_i approaches π . We will see that the resonance becomes incomplete as we increase the initial misalignment angle of a . This can be understood by noting that, when the anharmonic effect is important, the typical time scale around the end points of oscillations is longer than the mass scale around the potential minimum, which enhances the breaking of the adiabaticity condition.

B. Abundance

The number of the QCD axions in a comoving volume is an adiabatic invariant that is conserved during the conversion processes, if the adiabaticity parameter ξ is much smaller than unity, and if the anharmonic effect is negligible. Therefore the resultant energy density of the axion CDM is expected to be suppressed by the mass ratio, m_H/m_a , compared to the case without resonant conversion.

To see this, we have numerically solved the axion dynamics with such initial condition that only coherent oscillations of a_1 are induced while a_2 initially sits at the potential minimum. The results are shown in Fig 2. Here we have taken $F_a = F_H = 10^{14} \text{ GeV}$

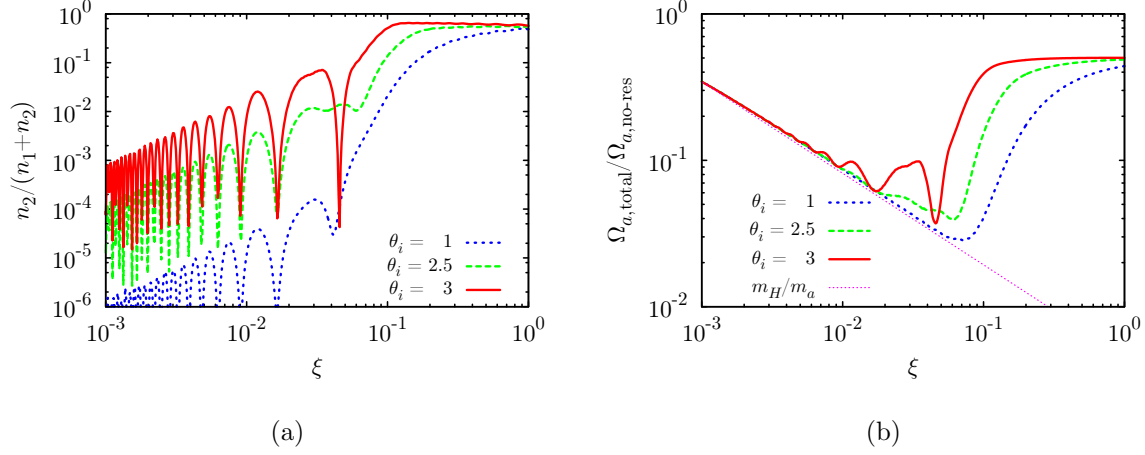


FIG. 2: The ratio of the number density of a_2 to the total number density (left panel) and the suppression factor of the axion density parameter with respect to the case without resonant conversion (right panel). The resonant conversion becomes efficient for small values of the adiabaticity parameter ξ . We have taken $F_a = F_H = 10^{14}$ GeV, and varied the initial misalignment angles as $\theta_i \equiv a_i/F_a = 1$ (dotted blue), 2.5 (dashed green) and 3 (solid red) with $\theta_{H,i} = -\theta_i$. The small dotted (magenta) straight line in the right panel shows m_H/m_a , and we can see that the resultant density parameter is indeed suppressed by the mass ratio for small $\xi \lesssim 0.1$.

and varied the initial misalignment angles as $\theta_i \equiv a_i/F_a = 1$ (dotted blue), 2.5 (dashed green) and 3 (solid red) with $\theta_{H,i} \equiv a_{H,i}/F_H = -\theta_i$. With this initial condition, a_1 starts to oscillate when the Hubble parameter becomes comparable to $m_a(T)$, while no coherent oscillations of a_2 are induced in the low energy, if the resonant conversion is 100 percent efficiency. In the actual Universe, however, the adiabaticity condition is weakly violated by a non-zero value of ξ , and as a result, a small amount of coherent oscillations of a_2 is induced. In Fig 2(a), we show the resultant number density ratio, $n_2/(n_1+n_2)$, as a function of the adiabaticity parameter ξ by varying m_H . Here the number density of each axion is defined by the energy density divided by the mass. One can see that only a tiny amount of a_2 is induced for small values of ξ , and the resonant conversion occurs efficiently. On the other hand, a larger fraction of a_1 is converted to the heavier eigenstate a_2 as ξ increases because of the incomplete resonant conversion. Also, one

can see that, as the initial misalignment angle θ_i increases, more a_2 is induced due to the incomplete conversion. This is because the anharmonic effect tends to break the adiabaticity condition, making the resonant conversion inefficient.

In Fig. 2(b) we show the resultant density parameter of the total axion CDM, $\Omega_{a,\text{total}} \equiv \Omega_{a1} + \Omega_{a2}$, normalized by the QCD axion density parameter in the absence of the resonant conversion, $\Omega_{a,\text{no-res}}$, as a function of ξ . As expected, the suppression factor is approximately given by the mass ratio, m_H/m_a , for small ξ , where the resonant conversion is efficient. Note that we actually vary m_H in this plot, while the other parameters are fixed. The suppression factor is of order 0.01 for the parameters adopted; we have chosen the decay constants that are slightly larger than the conventional axion window, since the numerical computation would become expensive, otherwise. For smaller values of F_a and m_H , the suppression factor m_H/m_a becomes smaller, and so, the final axion abundance can be suppressed by a larger amount. Thus, we can reduce the axion CDM abundance by making use of the resonant conversion of the QCD axions into hidden axions. Note that, although the initial misalignment angle for a_2 is set to be zero in our numerical calculation, the total axion abundance can be still suppressed for a certain range of the misalignment angle, because a_2 starts to oscillate before a_1 and its abundance tends to be suppressed compared to that of a_1 . Interestingly, the suppression factor exhibits oscillating behavior for ξ , which sensitively depends on the initial misalignment angle. This behavior affects the axion isocurvature perturbation as will be shown next.

C. Isocurvature perturbations

If the PQ symmetry is already broken during the last 50 or 60 e-foldings of inflation, the QCD axion acquires quantum fluctuations, leading to isocurvature perturbations imprinted on the CMB temperature anisotropy. Similarly, hidden axions also give rise to isocurvature perturbations. The amount of isocurvature perturbations is tightly constrained by the recent CMB observations [31]. Taking into account the anharmonic

corrections, the CDM isocurvature perturbation from both the QCD and hidden axions is calculated as [32]

$$\Delta_{\mathcal{S},\text{CDM}} = \left(\frac{\Omega_{a,\text{total}}}{\Omega_{\text{CDM}}} \right) \Delta_{\mathcal{S},a}, \quad (15)$$

with⁴

$$\Delta_{\mathcal{S},a} = \frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_i} \delta \theta_i + \frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_{H,i}} \delta \theta_{H,i}, \quad (16)$$

where Ω_{CDM} is the observed CDM density parameter, H_{inf} the inflationary Hubble parameter, and $\delta \theta_i$, $\delta \theta_{H,i}$ the quantum fluctuations of θ_i , $\theta_{H,i}$ with $\langle \delta \theta_i^2 \rangle = (H_{\text{inf}}/2\pi F_a)^2$, $\langle \delta \theta_{H,i}^2 \rangle = (H_{\text{inf}}/2\pi F_H)^2$. Assuming that there is no correlation between $\delta \theta_i$ and $\delta \theta_{H,i}$, the power spectrum of the isocurvature perturbation is given by⁵

$$\Delta_{\mathcal{S},a}^2 = \left(\frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_i} \right)^2 \left(\frac{H_{\text{inf}}}{2\pi F_a} \right)^2 + \left(\frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_{H,i}} \right)^2 \left(\frac{H_{\text{inf}}}{2\pi F_H} \right)^2. \quad (17)$$

The current upper bound on the CDM isocurvature perturbation reads [31]

$$\beta < 0.039 \quad (95\% \text{ C.L.}), \quad (18)$$

where β is defined by

$$\Delta_{\mathcal{S},\text{CDM}}^2 = \frac{\beta}{1 - \beta} \Delta_{\mathcal{R}}^2. \quad (19)$$

with $\Delta_{\mathcal{R}}^2 \approx 2.2 \times 10^{-9}$ being the curvature power spectrum. This sets stringent constraints on the inflation scale, which were studied in the literature in the case of only QCD axions [7–9].

In the absence of the resonant conversion, the abundance of QCD axions is a monotonically increasing function of the initial misalignment angle θ_i . In particular, it rapidly increases as θ_i approaches π , and so, $\Delta_{\mathcal{S},a}^2$ is also an increasing function of θ_i . This is

⁴ Here we have truncated higher order terms, which would be important only when the leading term is somehow suppressed. The effects of higher order terms are encoded in the non-Gaussianity of isocurvature perturbations, which we shall discuss later.

⁵ It is possible to modify the size of the quantum fluctuations if the radial component “saxion” evolves during the last 50 or 60 e-foldings [10, 33]. A non-trivial correlation between $\delta \theta_i$ and $\delta \theta_H$ can be generated if some combination of the axions was very heavy during inflation (cf. [13]).

the reason why the isocurvature perturbations get enhanced toward the hilltop initial condition [32, 34].⁶

If there is a resonant conversion of QCD axions into hidden axions, the situation is different. This is because the conversion rate depends on both θ_i and the mass ratio m_H/m_a . In Fig 3, we show $\Omega_{a,\text{total}}$ as a function of θ_i for various values of m_H/m_a . One can see that there is a plateau around $\theta_i \simeq 3$ for $m_H/m_a = 0.03$, where the isocurvature perturbations are expected to be significantly suppressed.⁷ In Fig. 4, we show the axion isocurvature perturbations normalized by the fluctuation of the misalignment angle of the QCD axion,

$$\left| \frac{\Delta_{\mathcal{S},a}}{\delta\theta_i} \right| = \sqrt{\left(\frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_i} \right)^2 + \left(\frac{F_a}{F_H} \right)^2 \left(\frac{\partial \ln \Omega_{a,\text{total}}}{\partial \theta_{H,i}} \right)^2}, \quad (20)$$

as a function of the adiabaticity parameter. One can see that there is indeed a significant suppression of the isocurvature perturbations at specific values of ξ and θ_i . The detailed structure of the suppression is shown in the right panel of Fig. 4; there are points where isocurvature perturbations are significantly suppressed. In the vicinity of these points $\partial \Omega_{a,\text{total}}/\partial \theta_i$ vanish and its sign flips between these points.

IV. DISCUSSION AND CONCLUSIONS

In this letter, we have studied the model of the QCD and hidden axions with a mass mixing and performed numerical calculations to follow the MSW-like conversion process between them, taking account of a weak violation of non-adiabaticity as well as the anharmonic effects. To characterize the violation of the adiabaticity, we have introduced an adiabaticity parameter, ξ , defined by the ratio of the Hubble parameter to the axion mass at the resonance (see Eq. (14)), and we have found that the resultant axion CDM

⁶ In addition, the non-Gaussianity is enhanced in the hilltop limit. [32].

⁷ Note that the axion abundance exceeds the observed dark matter for the adopted parameters, because $F_a = 10^{14}$ GeV is chosen for efficient numerical calculation. The purpose of this letter is to study the effect of the resonant conversion on the isocurvature perturbations, and we leave the calculation with smaller values of F_a for future work, as the required numerical computation is more expensive. Similar suppression is expected for the case with smaller values of F_a .

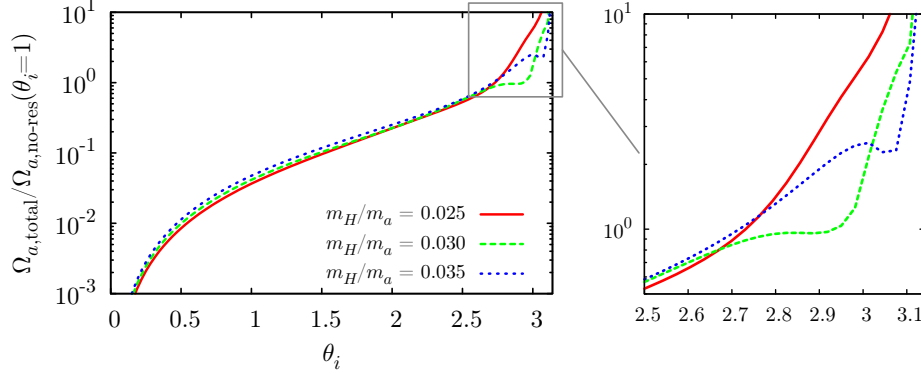


FIG. 3: The θ_i -dependence on the density parameter of the axion CDM is shown. The vertical axis is normalized by the density parameter in the single QCD axion case with $\theta_i = 1$. We have taken $F_a = F_H = 10^{14}$ GeV, $\theta_{H,i} = -\theta_i$ and $m_H/m_a = 0.025$ (solid red), 0.03 (dashed green), 0.035 (dotted blue).

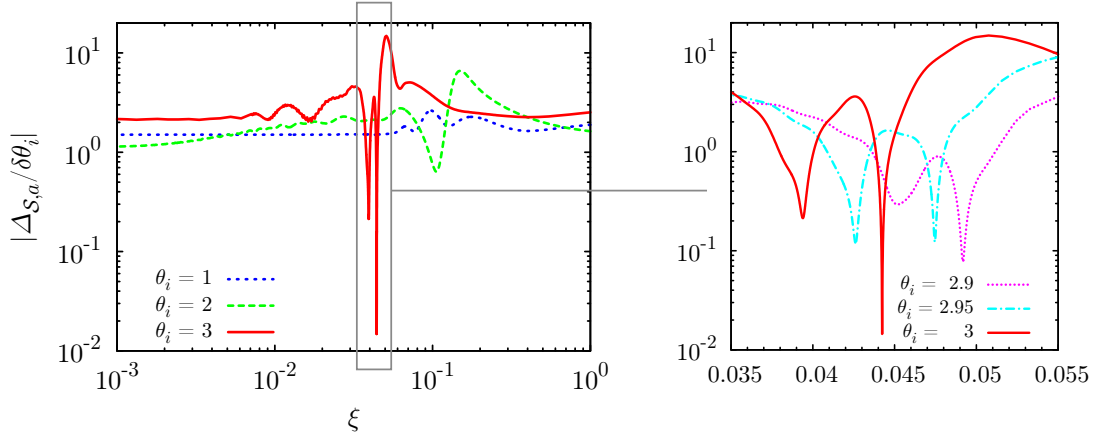


FIG. 4: The normalized isocurvature perturbation of the axion CDM, $|\Delta_{S,a}/\delta\theta_i|$ (see Eq. (20)), is shown as a function of the adiabaticity parameter defined in Eq. (14) for several values of the initial misalignment angle θ_i . We have taken $F_a = F_H = 10^{14}$ GeV, $\theta_{H,i} = -\theta_i$ and $\theta_i = 1$ (dotted blue), 2.5 (dashed green), 3 (solid red), 2.9 (small-dotted magenta) and 2.95 (dash-dotted cyan).

abundance can be suppressed by a factor of m_H/m_a if the resonant conversion is efficient, i.e., if the adiabaticity parameter is much smaller than unity and the anharmonic effects are negligible. Furthermore, we have found that the anharmonic effect makes the resonant conversion less efficient and, most interestingly, it significantly affects the axion CDM isocurvature perturbations. We have shown that the axion CDM isocurvature perturbations can be suppressed for certain values of the parameters where the resonant conversion is incomplete. In the following we mention on the limitations and implications of the present analysis.

We have performed the calculation with the initial condition that the hidden axion direction sits at the minimum, $\theta_i + \theta_{H,i} = 0$, in order to focus on the resonant conversion from QCD axions to hidden axions. For a more general initial condition, the resonant conversion from hidden axions to QCD axions also takes place, which complicates the dynamics of axions. In fact we have calculated such cases and confirmed that the isocurvature perturbations can be similarly suppressed for a certain range of the parameters. In order to estimate the suppression factor quantitatively, we need to scan the parameter space in the $\theta_i - \theta_{H,i}$ plane. Note that m_H needs to be much smaller than m_a to suppress the axion abundance, but the commencement of the hidden axion oscillations is delayed for small m_H and the amplitude may not be damped sufficiently at the resonance. Thus, there is a limitation to the suppression and we will investigate the conditions under which the total axion CDM abundance is maximally reduced.

Throughout the letter, we have taken $F_H = F_a$ for simplicity. If we choose a smaller value of F_a with $F_a < F_H$, the QCD axion mass at the zero temperature is increased, and as a result, the total axion DM abundance will decrease as it is suppressed by a factor of m_H/m_a . On the other hand, if we choose $F_a > F_H$, the hidden axion oscillations induced by the resonant conversion of the QCD axion can climb over the cosine potential hill and roll down to the adjacent minimum, $a_{H,\min} = 2\pi F_H$. We have confirmed this behavior numerically. This implies that domain walls may be formed by the resonant conversion. Once domain walls are formed and if they are stable, they will dominate the Universe soon and our present Universe cannot be realized. We will also study the parameter region to

avoid such a domain wall formation in the future.

So far we have considered only a Gaussian part of the isocurvature perturbations, but higher-order terms become important when the leading Gaussian part is suppressed. The constraint on non-Gaussianity of the isocurvature perturbations is characterized by the parameter, $f_{\text{NL}}^{(\text{iso})}$ and the current constraint reads [35]

$$\left(\frac{\beta}{1-\beta}\right)f_{\text{NL}}^{(\text{iso})} = 40 \pm 66. \quad (21)$$

While the exact form of $f_{\text{NL}}^{(\text{iso})}$ in the multi-axion case is rather involved, it is roughly estimated as $\beta f_{\text{NL}}^{(\text{iso})} \sim \partial^2 \ln \Omega_{a,\text{total}} / \partial \theta_i^2, \partial^2 \ln \Omega_{a,\text{total}} / \partial \theta_{H,i}^2$. We have also checked that the non-Gaussianity of the isocurvature perturbations are not suppressed in general, even if the Gaussian part is suppressed due to the incomplete resonant conversion.⁸ These second derivatives are of $\mathcal{O}(10 - 100)$ for the parameters of our interest, and the current constraints from non-Gaussianity of the isocurvature perturbation can be (marginally) satisfied. The mild enhancement of the non-Gaussianity is a necessary outcome of our scenario, because the anharmonic effect plays a crucial role in suppressing the power spectrum of the isocurvature perturbations (cf. footnote 6). It is worth studying how the resonant conversion affects non-Gaussianity, which we leave for future work.

We have focused on the resonant conversion between the QCD and hidden axions so far. In principle a similar resonant conversion could take place between two hidden axions with a mass mixing⁹, if one of them gradually acquires a mass and there appears a cross-over of the mass eigenvalues. The isocurvature perturbations of the hidden axions can be suppressed similarly for a certain set of the parameters.

⁸ Vice versa; we observe that the non-Gaussianity can be suppressed for a certain range of the parameters, where the Gaussian part is not suppressed.

⁹ This argument is not limited to axions, but, in principle, it can be applied to coherent oscillations of any scalars.

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